Elastic electron-deuteron scattering and two-body current operators in the Light-Front Hamiltonian Dynamics

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”Ingredients” from:

1. F. Lev, E. Pace and G.S., PRC **62**, 064004 (2000): *Deuteron FF’s in one-body Approx.*
2. T. Frederico,J.A. Marinho E. Pace, P. Sauer and G.S., PRD **77**, 116010 (2008): *Light-Front projection and the Ward-Takahashi identity*
3. J. De Melo, T. Frederico, E. Pace, S. Pisano and G.S., PLB 671, 153 (2009): *Nucleon FF’s in a microscopic VMD*

Outline

- Motivations and Issues
- Choosing the frame: Light-Front HD + Breit reference frame
- Projecting the em current onto the Light-Front hyperplane
- The Nucleon form factors in a Light-Front approach, or constructing a microscopic VMD
- Deuteron em observables: LF One-body + Two-Body contributions, preliminary results
- Summary & Perspectives

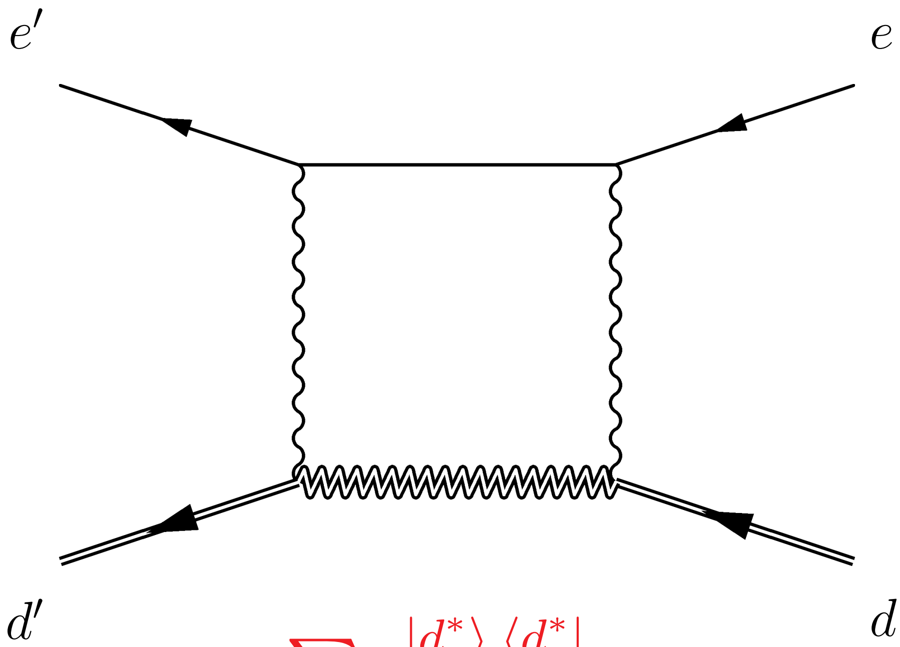
Motivations and Issues

- A careful description of the Deuteron, retaining only a finite number of baryonic and mesonic degrees of freedom, could help to single out new, experimental signatures, e.g. related to the underlying degrees of freedom. In the present context, ”careful description” means an approach that fulfills general properties, like the extended Poincaré covariance, Hermiticity and current conservation.
- Deuteron electromagnetic observables offer a valuable play ground for testing theoretical ideas
- The TJLAB upgrading opens very challenging and intriguing scenarios, as described, e.g., in PAC 34 and 35 Reports; therefore the theoretical tools should be made accurate as much as possible.

But...

- Consistency between dynamics and current operators.
- Strong interplay between different ingredients, as e.g., operatorial structure of the two-body currents and nucleon form factors.

- More degrees of freedom, like Δ?
- Two-photon exchange?



$$\sum_{d^*} \frac{|d^*\rangle \langle d^*|}{M_{d^*}^2 - P_{d^*}^2}$$

$$M_{d^*}^2 > 4M_N^2$$

Our present aim is to include two-body contributions to the em current, in a Light-Front approach, for describing the Deuteron em observables, still satisfying the extended Poincaré covariance.

NB: the present two-body terms are inspired by an exact analysis of the 4D current projected onto the LF hyperplane, where the projected current fulfills the Ward-Takahashi Identity (MFPSS, PRD 77, 116010 (2008))

Choosing the frame

★ Theoretical frame: the Light-Front Hamiltonian Dynamics, i.e. one of the Relativistic Dynamics proposed by Dirac in a seminal paper (RMP 21 (1949) 392). In general, RHD's allow one to rigorously fulfill the Poincaré covariance, for an interacting systems with a finite degrees of freedom. In some sense, RHD's fall between the non-relativistic quantum mechanics and local relativistic field theories.

Advantages in LFHD, among them : i) maximal number of Poincaré generators not affected by dynamics ii) $P^+ = P^0 + P_z \geq 0$; iii) sharp and clean separation between the CM propagation and the intrinsic one.

★ ★ Reference frame: a Breit frame where the momentum transfer is longitudinal, i.e.

$$\boxed{\vec{q}_\perp = 0}$$

The symmetry of the process allows one (LPS, NPA 641, 229(1998)) to reduce the constraints imposed by the "extended" Poincaré covariance to a simple rotational covariance around the $\hat{z} \equiv \hat{q}$ for an auxiliary current operator that i) depends parametrically upon the CM momenta and ii) acts on the intrinsic variables. Any operatorial dependence fulfilling the rotational covariance around the $z - axis$ is allowed. (WTI....)

Projecting the em current onto the Light-Front hyperplane

By using two ingredients: i) the Quasi-Potential approach for the Transition matrix (Woloshyn -Jackson NP B64, 269 (1973)), and ii) the projection of the 4D physical quantities onto the 3D Light-Front hyperplane (i.e. $x^+ = x^0 + x^3 = 0$) (Frederico et al PRD (2000)-(2008)), one can establish a formally exact correspondence between

- 4D Bethe-Salpeter amplitude and the 3D Light-Front "valence" wave function.
- matrix elements of the 4D em current and a 3D LF current, that fulfills a Ward-Takahashi identity.

Onto the 3D LF plane, one is left with the dependence upon the relative-time propagation ($\longrightarrow$ intrinsic dynamics).

At the end, 3D, dynamical eqns with an effective 3D interaction, exactly related to the 4D kernel of the BS Eqn, are found

A possible approximation scheme for constructing solutions is based on the Fock expansion of the relevant quantities (work in progress for non perturbative solutions). Then, one has both dynamical eqns (effective interaction) and current operators (fulfilling the WT identities) that contain Fock components up to a given order, as can be seen through a time-ordered analysis of the diagrams associated to each contribution.

★ One could argue that the convergence rate of the QP expansion is related to the smallness of the probability for the higher Fock-components.

★ ★ Explicit expressions for many-body terms of the em 3D LF current has been obtained, within a Yukawa model in ladder approximation for an interacting system of two fermions (MFPSS PRD 77 (2008))

In the actual calculation, there is another relevant issue for the LF projection, that makes the application of the procedure to the fermionic case sharply different from the bosonic one. If in the fermion propagator the on-shell term is separated out

$$iS(k) = \frac{\not{k} + m}{k^2 - m^2 + i\varepsilon} = \frac{\not{k}_{on} + m}{k^+(k^- - k^-_{on} + \frac{i\varepsilon}{k^+})} + \frac{\gamma^+}{2k^+}$$

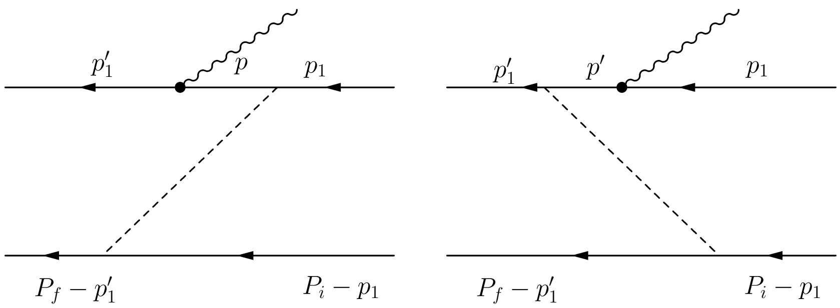
where $k^-_{on} = (\mathbf{k}^2_\perp + m^2)/k^+$ is the minus-component of k^μ_{on}, such that $k_{on} \cdot k_{on} = m^2$, then the second term leads to an instantaneous (in LF time !!) free propagation, as shown by a Fourier transform ($\int dk^- exp(-ik^- x^+/2)$) leads to $\delta(x^+)$.

The instantaneous term has a great impact on the analytic structure of the 4D matrix elements of the current operator, and it produces very peculiar operatorial structures in the many-body contribution to the 3D LF current.

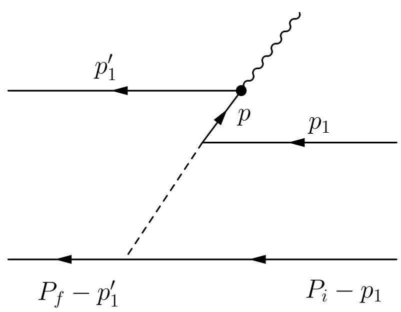
Two-Body Current in the LF approach: Yukawa model in ladder appr.

First order em current (zero order is the one-body current). A time-ordered, diagrammatic analysis allows one to appreciate the physical content in Fock states, inside the operator.

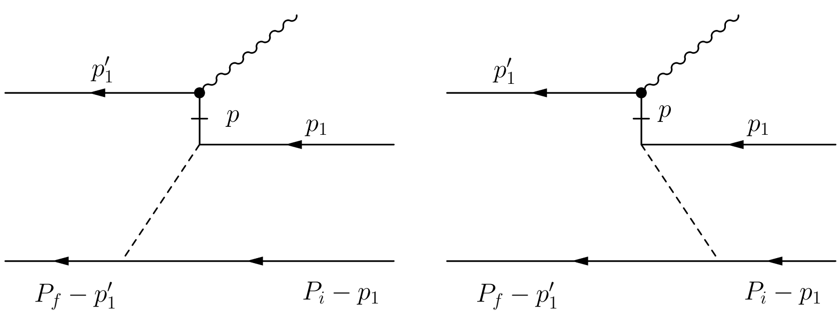
One has at least three particles in flight (LF-time flows $R \to L$)



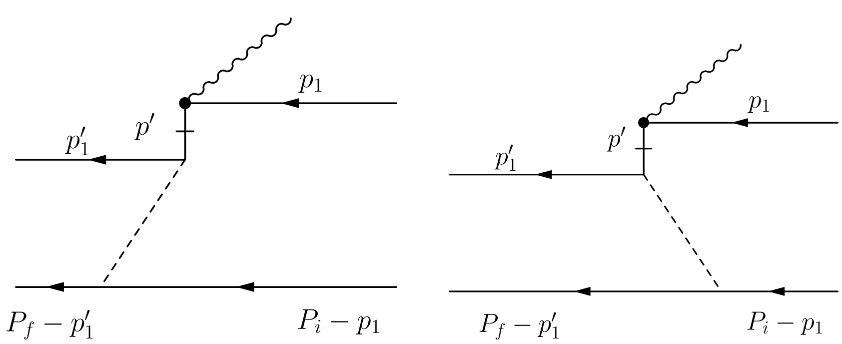
Interaction terms



Pair term



Instantaneous, in LF time, terms



Deuteron em observables and two-body contributions in LFHD

In a Breit frame where

$$\boxed{\mathbf{q}_\perp = 0} \Rightarrow \boxed{q^+ \neq 0}$$

the matrix elements of the em current operator for an **Interacting System** can be defined in terms of the 3D LF current and the $\perp$ component of $\vec{S}$ (dynamical generators in LF) as follows

$$j^\mu(q\hat{e}_z) = \frac{\mathcal{J}^\mu(q\hat{e}_z)}{2} + \overbrace{L^\mu_\nu[r_x(-\pi)]\, e^{\imath\pi S_x}\, \frac{\mathcal{J}^\nu(q\hat{e}_z)^*}{2} e^{-\imath\pi S_x}}^{\text{from Hermiticity}}$$

with

$$\mathcal{J}^\mu(q\hat{e}_z) = j^\mu_{one} + j^\mu_{two} = \overline{\Pi}_0 \mathcal{J}^\mu_0 \Pi_0 + \overline{\Pi}_0\, [V \Delta_0 \mathcal{J}^\mu_0 + \mathcal{J}^\mu_0 \Delta_0 V]\, \Pi_0$$

where i) $\Pi_0 \equiv$ *free LF projector*, ii) $\Delta_0 \equiv G_{free} - G_{glob}$, iii) V is the interaction mediated by a chargeless pion

$$V = i\, g^2\, \tau_{13}\, \tau_{23}\, \gamma^5_1 \otimes \gamma^5_2\, \mathcal{F}[(\hat{p}_2 - \hat{p}_1)^2] / \left[(\hat{p}_2 - \hat{p}_1)^2_- m^2_\pi + i\epsilon\right]$$

and iv)

$$\mathcal{J}^\mu_0 = \textstyle\sum_{i=1,2} J^\mu_{pi}(0)(1+\tau_3)/2 + J^\mu_{ni}(0)(1-\tau_3)/2$$
$$J^\mu_N = -F_{2N}(p^\mu + p'^\mu)/2M + \gamma^\mu(F_{1N} + F_{2N})$$

★ j^μ contains a many-body contribution even retaining only j^μ_{one}, given the presence of the dynamical operator S_x

What about current conservation and charge normalization?

In the chosen Breit frame, CC and CN impose

$$\langle P_f, d|j^+(q\hat{e}_z)|d; P_i\rangle = \langle P_f, d|j^-(q\hat{e}_z)|d; P_i\rangle$$

$$\langle P_i, d|j^+(0))|d; P_i\rangle = \langle P_i, d|\frac{1}{2}\left[\mathcal{J}^+(0) + \mathcal{J}^-(0)\right]|d; P_i\rangle = e$$

If WTI is fulfilled then one has CC, once matrix elements are taken between eigenvalues corresponding to the interaction V. This is not the case in our phenomenological calculations, since we adopt the Deuteron wave function corresponding to realistic interaction, like AV18, CD-Bonn etc.

But, in the elastic processes, CC follows after implementing Hermiticity, given by

$$\langle P_f, d|j^\mu(q\hat{e}_z)|d; P_i\rangle = \langle P_i, d|j^\mu(-q\hat{e}_z)|d; P_f\rangle^*$$

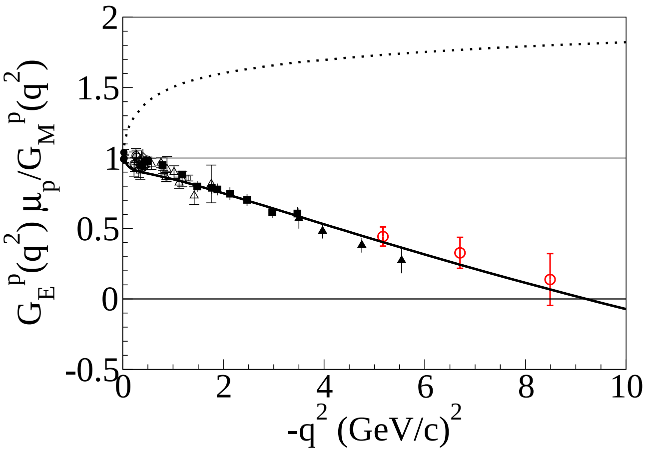
The charge normalization can be fulfilled if :
$\langle P_i, d|\mathcal{J}^-(0)|d; P_i\rangle = \langle P_i, d|\mathcal{J}^+(0)|d; P_i\rangle$

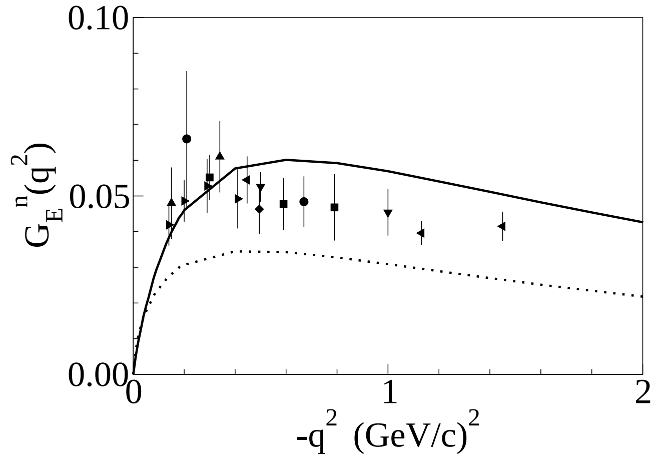
★ This leads to assume such an equality for any q

Note: for evaluating of the em observables only $j^+(q\hat{e}_z)$ and $j^{1(2)}(q\hat{e}_z)$ are relevant.

★★ The extended Poincaré covariance is fulfilled (LPS (2008))

Nucleon form factors in a Light-Front approach

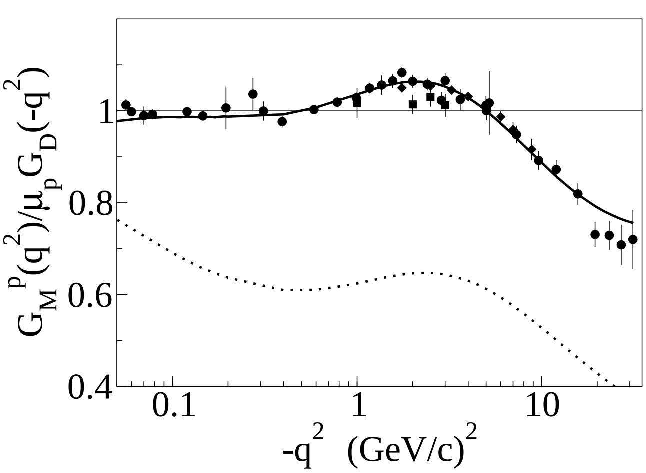


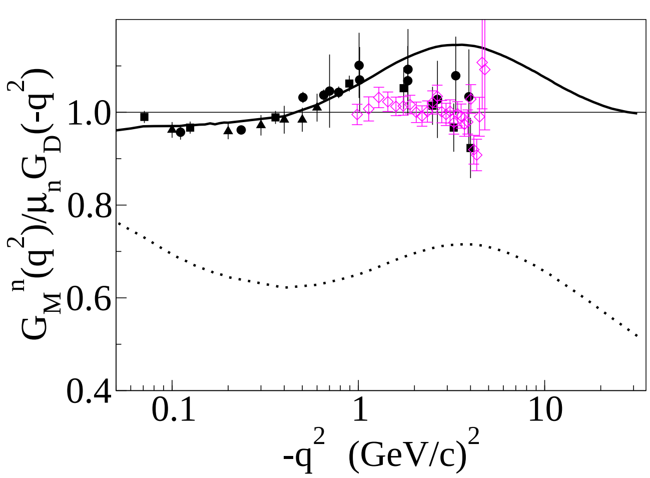


Data: www.jlab.org/ cseely/nucleons.html and Refs. therein, ○ latest data from TJLAB, A. J. R. Puckett et al, arXiv:1005.3419

4 adjusted parameters - from $G_M^{p(n)}$ and G_E^n

The possible zero in $G_E^p \mu_p / G_M^p$ is strongly related to the Z-diagram contribution, i.e. higher Fock components.





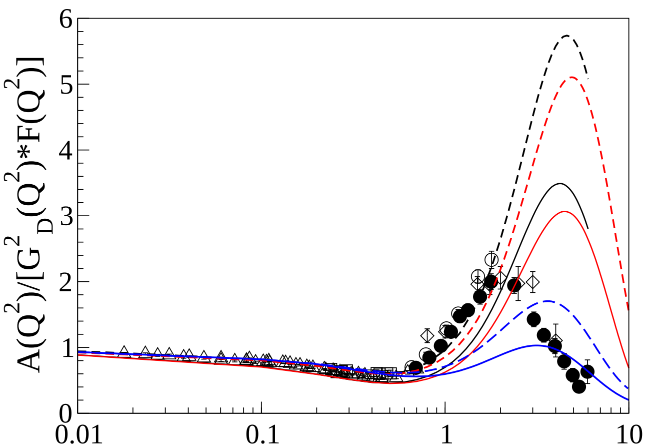
◇: J. Lachniet et al PRL 102, 192001 (2009)

Solid line: full calculation $\equiv \mathcal{F}_\triangle + [Z_B\, \mathcal{F}_{bare} + Z_{VM}\, \mathcal{F}_{VMD}]_{q\bar{q}}$

Dotted line: $\mathcal{F}_\triangle$ (triangle contribution only)

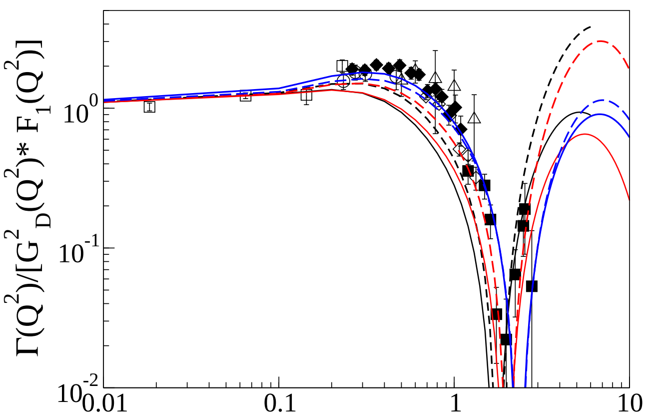
$G_D = 1/[1 - q^2/(0.71\,(GeV/c)^2)]^2$

Deuteron em observables in free-current approximation

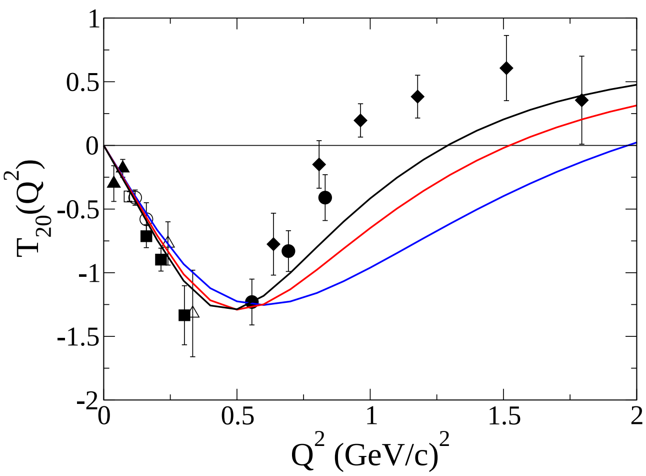


$$A(Q^2) = G_C^2 + \frac{8}{9}\tau^2 G_Q^2 + \frac{2}{3}\tau G_M^2$$

with $\tau =_Q^2 /4M^2$



$$B(Q^2) = \frac{4}{3}\tau(1+\tau)G_M^2$$



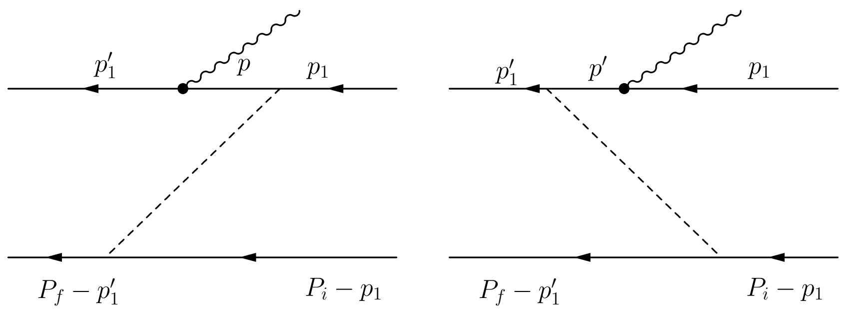
$$T_{20}(Q^2) = -\tau \frac{\sqrt{2}}{3} \times$$

$$\frac{[\frac{4}{3}\tau G_Q^2 + 4G_Q G_C + f G_M^2]}{A + B\tan^2(\theta/2)}$$

Solid lines : CD-Bonn + LF-FF, AV18 + LF-FF, RSC93+ LF-FF. Dashed lines: LF-FF $\longrightarrow$ Gari-Krümpelman-FF

Preliminary results with 2B terms

To start, we have considered the interaction terms



Interaction terms

for evaluating magnetic and quadrupole moments

Interaction	P_D	μ_D^{NR}	μ_{one}^{LFD}	μ_{1+2}^{LFD}
CD-Bonn	4.83	0.8523	0.8670	0.863 ± 0.002
RSC93	5.70	0.8473	0.8637	0.861 ± 0.002
Av18	5.76	0.8470	0.8635	0.860 ± 0.002

Exp. 0.857406(1)

Interaction	P_D	Q_D^{NR}	Q_{one}^{LFD}	Q_{1+2}^{LFD}
CD-Bonn	4.83	0.2696	0.2729	in prg.
RSC93	5.70	0.2703	0.2750	in prg
Av18	5.76	0.2696	0.2744	in prg

Exp. 0.2859(3)

From the charge normalization, one can obtain the probabilities of the valence and non valence components

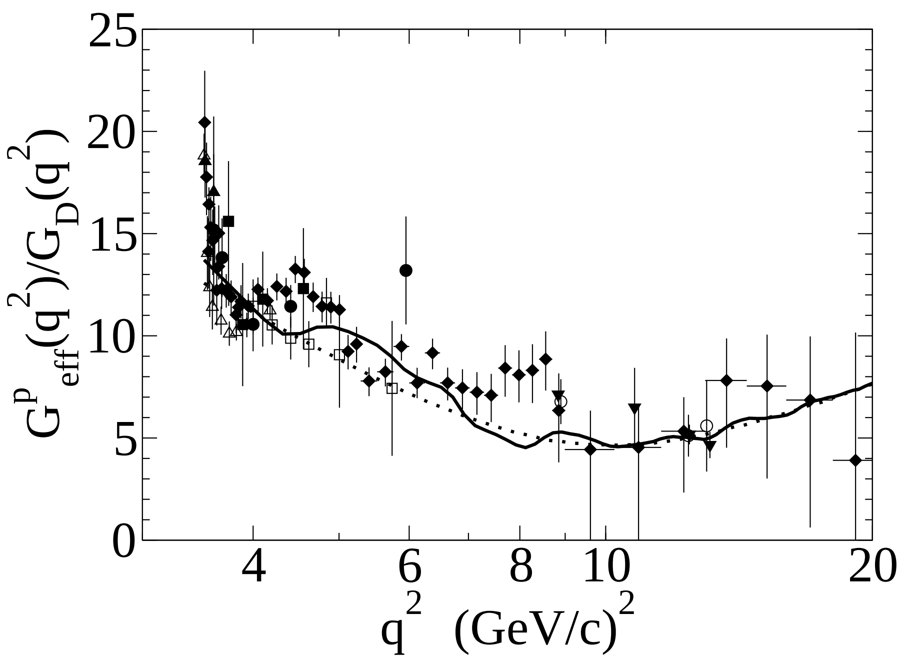
At the present stage we have obtained $Prob_{NV} < 0.01$

Summary & Perspectives

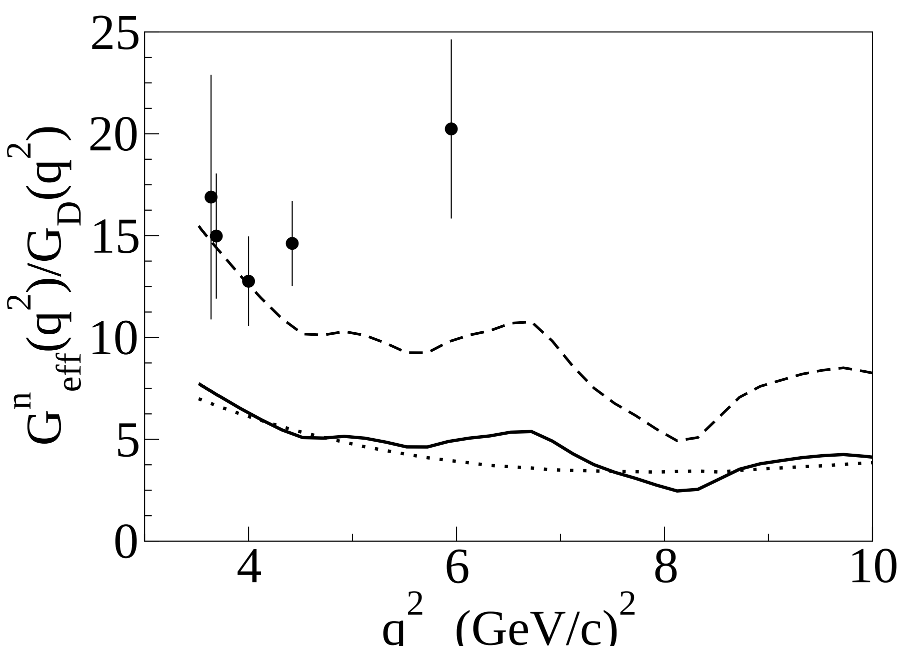
A first summary: an attempt to include two-body, dynamical contributions to em current, within LFHD has been undertaken

- The approach is fully Poincaré covariant
- The two-body contributions, adopted in the calculation of the Deuteron em observables, have been inspired by an exact analysis of a Yukawa model for two interacting fermions in ladder approximation. It turns out that onto the LF hyperplane one obtains a 3D current fulfilling the Ward-Takahashi identity, at each order in the Fock expansion
- The systematic analysis of the Dueteron em ff's is started, and it represents a non trivial task from the numerical point of view, given the many, multidimensional integrals to be performed with very high accuracy, in particular at low q^2.
- First results for CD-Bonn, RSC93 and AV18 interactions with the interaction contribution (exchange a chargeless pion) close to the photon point appears consistent with the expectation: very low probability for the component beyond the valence one, and magnetic moment in fair agreement with the experimental values. Quadrupole moments are coming soon....

★Nucleon timelike form factors: parameter-free results



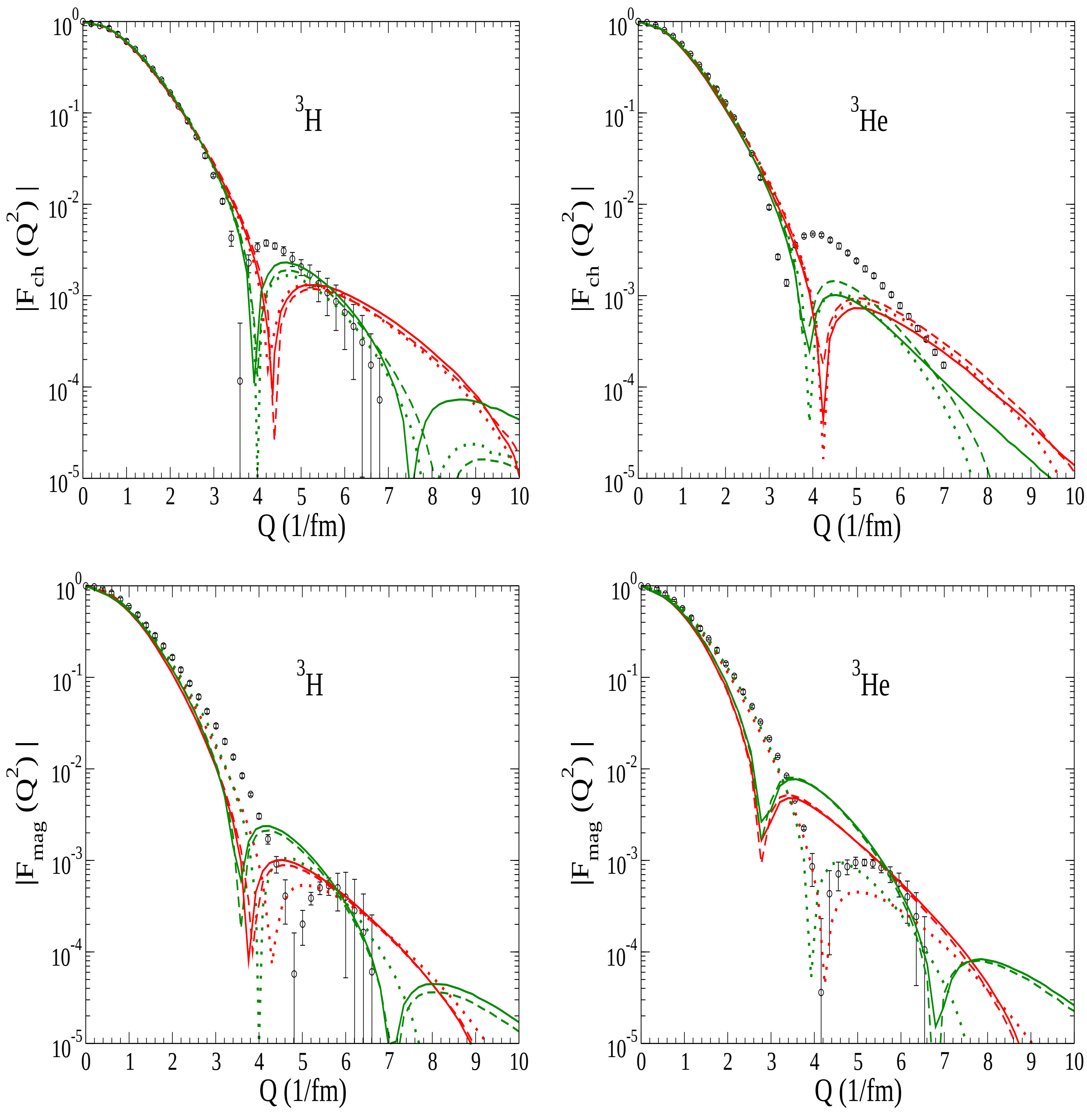
Missing strength at $q^2 = 4.5\ (GeV/c)^2$ and $q^2 = 8\ (GeV/c)^2$



$$G^2_{eff}(q^2) = \frac{|G_M(q^2)|^2 + |G_E(q^2)|^2 \ \frac{2m_N^2}{q^2}}{1 + \frac{2m_N^2}{q^2}}$$

Charge and Magnetic form factors of ^{3}H and ^{3}He in a frame where $\boxed{\mathbf{q}_\perp = 0}$, and AV18 Two-body forces + Coulomb

For the first time in LFD !, but without two-body dynamical currents



Solid line: LF full calculation $(S + S' + P + D)$ and LF Nucleon ff's; dashed line: LF full calculation and Gari-Krümpelman N ff's; dotted line: $S + S'$ waves + GK.

In Green color, NR calculations